The medium is the message: measuring area with different media

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Abstract

The use of concrete media for developing mathematical understandings has been an unquestioned aspect of mathematics education for many years. Whilst the efficacy of such materials for most children is accepted, the different understandings fostered by different media is less well defined. This paper reports an investigation into the various responses of a large sample of eight-year-old children who used three different media for measuring the area of a rectangular space. Discussion focuses on commonalities and diversities revealed.

Introduction

In 1995 the Victorian Board of Studies contracted the Australian Council for Educational research to prepare materials for the Learning Assessment Project (LAP) statewide assessment in mathematics and English. The guiding structure for the content of the mathematics assessment was to be the Curriculum and Standards Framework (CSF) in mathematics for Victorian schools (Board of Studies, 1995), which is based on the so-called 'national profiles' in mathematics (AEC, 1994). Not only does the CSF gives an overview of the content expected to be taught in schools, but also it advances an approach to the teaching of mathematics that could be described as generative; children are encouraged to be active generators of the mathematics they study, not merely passive recipients of adult 'wisdom'.

The items developed for assessment of the space and measurement aspects of the CSF at Year 3 (8-year-olds) were contextualised in a short story based on 'Goldilocks and the Three Bears'. A number of these items were performance-based assessment items; that is, the student responded by performing a mathematical task, such as measuring length or constructing a 3-dimensional model. It is one such item, the measurement of a rectangular area, that is the focus of this paper.

Related literature

The CSF lists *inter alia* the outcome that students at this level '...use direct comparison (for example, by placing on top of) and uniform units (for example, same size tiles) to order areas...' (CSF: 62). This outcome was considered to be suitable as a basis for a performance item that required children to actually measure the area of a rectangular shape with some appropriate concrete medium. Figure 1 illustrates the item.

Since the LAP items were to be standardised, the same material had to be readily available to all children across the state. The issue that confronted the

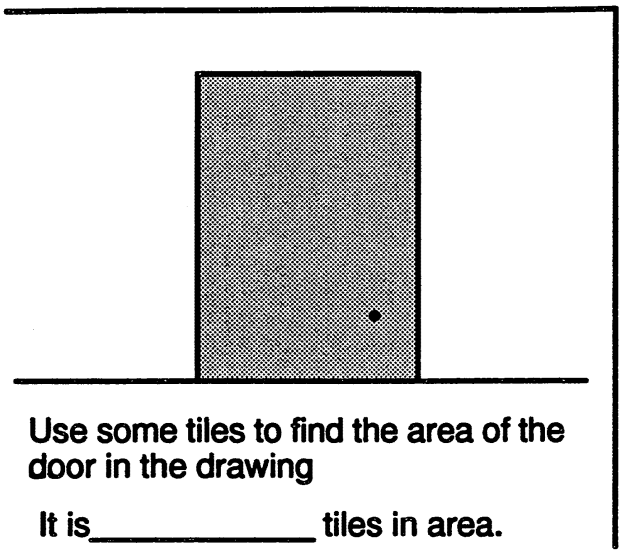

Figure 1: The area of the door item from 'Goldilocks and the Three Bears'.

LAP development team was exactly which medium should be used. The research literature dealing with the use of concrete referents, where this research exists, focuses on either the necessity to use these materials, or on how teachers employ them in the classroom. As Hart points out (Hart, 1993) the common view is that '...the use of concrete materials is good in itself irrespective of the topic being taught.' (page 44). Further, Hart quotes Ginsburg and Opper who suggest that Piagetian theory requires that the child '... must physically act on his environment. Such activity constitutes a major portion of genuine knowledge ...' (Hart, 1993: page 44).

With respect to which medium might be the most efficacious for learning a particular aspect of mathematics, the research literature is curiously silent.

Curriculum documents from several Australian state education systems on the other hand, suggest a wide variety of materials that could be used for measuring area at this level; envelopes, cups, cans, and Base 10 materials (New South Wales Department of Education, 1989); leaves, and lids (Department of Education and the Arts, Tasmania, 1994); sheets of newspaper, and tiles (Department of School Education Victoria, 1992: Board of Studies, 1995). There were two constraints in the selection of a particular medium: its universal availability, since the standardised nature of the LAP assessment required that all children have access to the selected medium, and the need for a uniform, tessellating unit. Leaves, lids and so forth were considered unusable because of these criteria. The remaining options, Base 10 blocks and tiles were therefore considered. Curriculum document recommendations notwithstanding, and after much discussion, Base 10 blocks were rejected on the grounds that they were three-dimensional and thus could only be used for measuring volume. This left tiles as the preferred medium.

In the state-wide implementation of the LAP items, light card 'press-out' tiles were to be provided (to assure standardisation and availability) but for the trial of the items, tiles were printed on paper, and the children were to use scissors to cut them out before use.

Sample

The trialling of the LAP items in mathematics was conducted in a socio-economic cross-section of urban and suburban government and Catholic schools. The trial schools used a wide variety of curriculum materials and resources and included many different teaching situations and styles. As such they were representative of schools across the nation.

Design

The design is not a research design in the traditional sense, but rather an adaptive design sited within an action research paradigm. The history of its development will make this clearer.

The trialling of LAP items was to be conducted over three weeks; at the end of the first week, when 199 children had measured area with paper cut-out tiles, it was apparent that these tiles were not satisfactory. There were difficulties for 8-year-old children in cutting regular, uniform squares, and problems such as

overlapping of the measuring unit (forming a sort of collage) were causing concern. For the second week of trialling the paper tiles were replaced with Base 10 one-centimetre cubes that were readily available in trial classrooms. The dimensions of the area to be measured were adjusted to this new unit in order to maintain the comparability of responses with the paper tiles used earlier. The cubes caused new anomalies (for example children building three-dimensional structures) and at the end of the week after 173 children had used them, these too were replaced. It was decided to try a medium 'between' the previous two to gain some understanding of the effects of various media, particularly those with different thickness. Wooden tiles about three millimetres thick were purchased as these were neither thin and capable of being overlapped, nor were they as thick as building blocks. In all forty-eight children used these wooden tiles to measure an (again) modified area.

This unbalanced sample does not provide complete generalisability, but the range of the data collected would appear to indicate that we have mapped a large part of children's thinking about the measurement of area with these media.

Data collection and analysis

The item under discussion was not singled out for special treatment during the testing period; no hints as to how to measure area were given, nor any reference made to 'covering' or other paraphrases. Piloting had brought to light several commonly occurring responses and these were used to define assessment categories. Children's responses were assigned to an assessment category and this category recorded. Responses that did not fit an existing category were recorded (by a written description, drawing or photograph). In this way a record was produced of every child's response. Table 1 sets out the percentage of children giving each response for the three media used. All percentages have been rounded to one decimal place, and a '–' indicates no responses in that category.

Table 1: Categories of response, by medium used.

Response	*Paper tiles (n = 199)* %	*Cubes (n = 173)* %	*Wooden tiles (n = 48)* %
Correct area	38.2	45.7	91.6
Area concept (minus door handle)	4.5	–	–
Area concept (bits of tiles)	3.5	–	–
Area concept (incorrect measure)	13.1	–	–
Area concept (collage)	11.6	–	–
Correct units (incorrect area)	0.5	–	–
Perimeter (exterior)	4.5	8.7	4.2
Perimeter (interior)	3.0	7.4	–
Length (door base)	0.5	0.6	2.1
Length (door height)	1.5	2.9	–
Length (other)	0.5	2.9	–
Pattern	3.5	0.6	2.1
Outline drawing	–	8.7	–
3-D construction	–	1.7	–
Other	1.0	–	–
No answer	15.1	20.8	–

– *indicates that n = 0*

Results for each medium

Paper tiles

One hundred and ninety-nine children used paper tiles as units to find the area of the door in the drawing. The children's responses fell into four broad categories; those which exhibited some understanding of the attribute of area, those which measured a perimeter, those who measured a length and those which made a pattern on the page.

About 38% of children completed the item correctly by covering the entire rectangle with the paper tiles, with only two children measuring length and width and multiplying or serial counting. Responses which exhibited some understanding of the attribute of area displayed a range of thinking that is interesting to examine. About 33% of children's responses displayed some partial understanding of the concepts involved in the item. Whilst these children showed an awareness of the attribute of area they used the paper tiles in unexpected ways. For example, some children covered the door but left the doorhandle uncovered (4.5%).

Paper being an easily cut medium enabled children to manipulate the unit in a variety of ways. It was this manipulation of the unit which revealed the variety of children's thinking about area and its measurement. In all more than 30% of the children displayed some understanding of area, whilst not being able to correctly measure the door area. Some carefully covered within the boundary of the door region but created units of varying sizes, whilst 13.1% used the given units inaccurately. This last group includes those who cut the tiles as strips, a problem with the way in which the tiles were presented. Other children made a collage (11.6%), that is they overlapped the tiles to cover the door using as many tiles as were needed to ensure no door showed through the tiling. These children showed an awareness of the attribute of area, but no understanding of the use of tiles as a unit of measure. A small number of children (3.5%) cut the tiles into 'bits' of units to cover spaces in three different ways; some filled the door region with the tiles, then noticing a gap, used a 'bit' to cover it over. They then ignored the 'bit', counting only the whole tiles used, showing the beginnings of ideas about units of measurement. Others cut the tiles into 'bits' and counted each 'bit' as a unit. The last group were those children who divided the tiles into fractional parts and summed these fractional units to find the total area.

Confusing perimeter with area is a well-known problem with young children, but of this sample only 7.5% did so; a further 2.5% measured a length, such as the base of the door. These children could be said to have no understanding of area. Some children (4.5%) created a pattern on the page. These children probably know that area is not length and perhaps that area has some two-dimensional quality, but they are at a very early stage of understanding the attribute. Fifteen per cent of children offered no response to the item despite adequate time being allowed for the completion of the test.

Wooden tiles

Thin, square tiles were offered to 48 children as the medium for measuring the area of the rectangular door. The use of thin tiles for measuring area represents the commonest experience of area measurement for most children. Whilst many media are used for developing the measurement of area concept, square tiles, usually of wood or plastic, are commonly used as the standard unit for practical measurement purposes prior to the introduction of the centimetre square unit. Familiarity with this medium could well be the reason for the high percentage of correct responses (some 90%).

There are alternative causes for the high correct response rate. Tiles, even though relatively thin, nevertheless

have sufficient thickness to stop them being overlapped. This means that covering a surface without overlapping is not an issue whereas abutting the tiles could be. The fact that the tiles and the area to be measured are commensurate essentially forces 'no gaps' if the area is to be 'covered' exactly.

The incorrect responses provided by this medium fall into two main categories. The first category is of responses that show children's well-known confusion between perimeter and area, although only a small percentage fell prey to this (about 4 %). Two children offered unusual responses; one measuring only the base (width) of the door, the other using a chequerboard pattern to 'cover' the door area. This latter child counted only those tiles in the pattern, not both tiles and spaces. Every child gave a response to the item.

Cubes

Dienes' MAB 'minis', one centimetre sized cubes are a familiar sight in classrooms, where they are used for a variety of purposes, including measuring length and area. Nearly half (45.7%) of the children who were provided with this medium for measuring the area of the door were able to find the area correctly. As one would expect in using a solid, three-dimensional object, there were no responses involving overlapping or the creation of 'bits'.

Predictably a group of children measured the perimeter, either exterior (8.7%) or interior (7.4%). The remaining children either measured a length, such as the height of the door (2.9%) or some combination of lines upon the page (2.9%). In all 22.5% of the children provided responses based upon a linear measure. This may well be due to the use of cubes as a unit for early experiences in measuring lengths, a use often suggested in curriculum materials.

Cubes are of course blocks, and 10.4% of the children used them to build a model of the door. Surprisingly a large group (20.8%) gave no response at all to the item despite the adequate time allowed.

Comparisons of the three media

A key difference between the media examined is in the proportion of children who can correctly measure area. Nearly twice as many children could measure correctly with the wooden tiles compared with either of the other two media. There are several reasons that can be suggested for this; tiles are the most familiar of the media commonly used in classroom area activities; they are not usually used for length measuring; tiles are rectilinear as was the area to be measured; they do not allow overlapping; they encourage 'no gaps'; as in this case they were commensurate with the dimensions of the door. Some of these qualities are shared by the cubes (for example, overlapping not possible and commensurability). However, the use of cubes for length measurement, combined with their 'building block' appearance, appears to encourage a large number of 'linear' responses. In short one can explain the variety of responses provoked by tiles and the cubes. What is happening with the paper tiles on the other hand is less clear; paper tile responses cover aspects of both tiles and cubes, a most unexpected occurrence. The similarity between paper and wooden tiles explains some responses, but the responses shared by cubes and paper tiles is puzzling.

Discussion

Looking at area measurement from a teaching and learning perspective the results of this investigation show that there is indeed a 'best' medium. It is clear that the use of wooden tiles as a medium for measuring area far surpasses other media considered here. However if one wishes to explore children's understanding of area and area measurement then clearly paper tiles are the 'best' medium. The results of this investigation show that the use of paper tiles elicits a wider range of thinking

than either of the other media. While the confusion between perimeter and area is elicited by all the media studied, paper tiles uncovered some, albeit infrequently occurring, unusual misunderstandings. These misunderstandings are critical for those children involved; for example, those who think it sufficient to cover the area, without regard for overlaps or gaps, could not be said to appreciate area in the formal sense. Yet their misunderstanding is masked by the use of three-dimensional media. Paper tiles provide us with the opportunity to see the range of ideas that children have about units. There are those who understand that parts of units can be used, and that these are fractions of the unit, as distinct from those who use parts of the unit, but either count the parts as units or disregard them entirely.

In summary there are three major conclusions that can be drawn; for teaching and learning wooden tiles appear to be the best medium, with fewer distracting aspects; for diagnostic investigations paper tiles would appear to be the best medium; and finally, there is a need for further work to be done in identifying what are the pitfalls associated with the use of particular media in mathematics learning.

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